Specification tests in DEA

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Data Envelopment Analysis

- DEA is a non-parametric technique used to evaluate the performance of comparable decision making units. It assumes that all deviations from the optimal level of production are due to inefficiency (there is no allowance for random error).

- Farrell (1957) in his seminal paper on “The Measurement of Productive Efficiency” introduced a tool to measure the comparative efficiency of productive units that use multiple inputs to produce several outputs.

- More than twenty years later Farrell’s work was developed by Charnes *et al.* (1978) who specified a linear mathematical program that could be used to choose the most efficient DMU amongst its peers.

Firm behaviour

- Traditional microeconomic analysis considers 2 inputs and 1 output.
- The production function f(x1,x2) denotes the maximum output y that can be obtained with given quantities of inputs x1 and x2.
- In most cases, it is assumed that

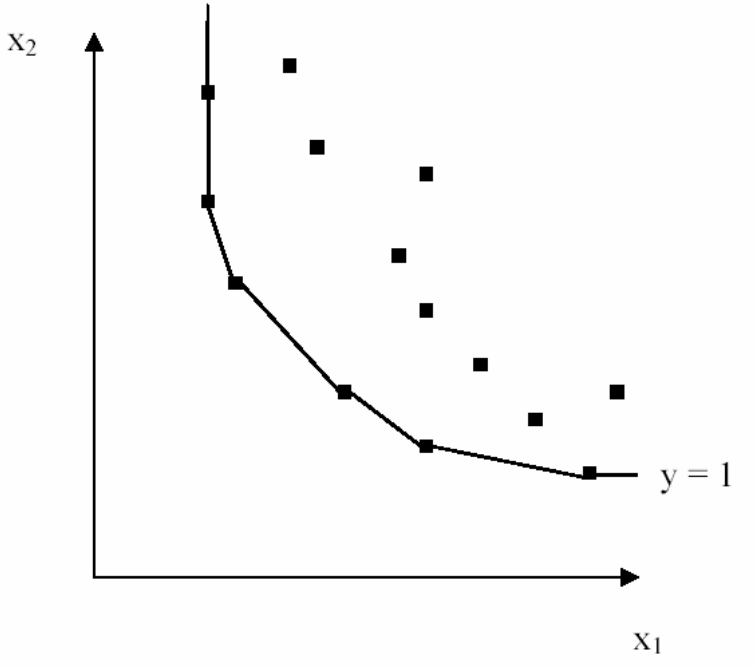
  $\partial f_i(x_1,x_2) / \partial x_i \geq 0$.

Firm behaviour

- The production function is a representation of the maximum attainable output: $0 \leq f(x1,x2) \leq y$.
- So, a firm is technically efficient if $y = f(x1,x2)$ and inefficient if $y < f(x1,x2)$.
- The ratio $y/f(x1,x2)$ is therefore a natural measure of efficiency. In fact it is the structural idea behind either Farrell's measurement of efficiency or the DEA method.
- As Farrell pointed out, the problem then is one of specifying a production function in practical terms. So, there are two main options to specify a production function: "a theoretical function specified by engineers and an empirical function based on the best results observed in practice".

Farrell

- The efficient isoquant obeys to two assumptions:
  - it is convex to the origin;
  - its slope is non-positive.
- So, in order to maintain the level of efficiency, an increased use of any input may not lead to a lower level of output (i.e., the marginal product of any input may not be negative).



Conclusions from Farrell's work

- If a DMU increases its consumption of a given input, it should increase the output to maintain the same level of efficiency.

- If a DMU increases its production without any extra consumption, its efficiency score increases.

DEA

- The DEA model (Charnes *et al.*, 1978) calculates the weights that maximize the ratio of outputs to inputs, taking into account that they should be non-negative and that for the chosen weights no DMU should have an efficiency ratio higher than 1.

$$\max h_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}$$

subject to

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \quad \text{with j = 1, …, n}$$

$$v_r, v_i \geq 0 \quad \text{with r = 1, …, s and i = 1, …, m}$$

DEA

- Assuming positive weights, the isoquants obtained with DEA also imply that higher use of an input leads to more output (in order to maintain the efficiency score).
- So, unlike it is usually assumed, the implicit production function on DEA has one important restriction: the marginal productivity of the inputs must be positive.

Implication

- Standard statistical tests might be used in order to check if the inputs and outputs selected for the analysis comply with this restriction avoiding the traditional reliance on researchers' choice.

Relevance of this implication

- A review of the papers published on DEA efficiency analysis applied to primary care (listed by Hollingsworth et al., 1999; Hollingsworth, 2003; Hollingsworth, 2008) shows that in several papers it is not clear if this implication is taken into account.

Examples

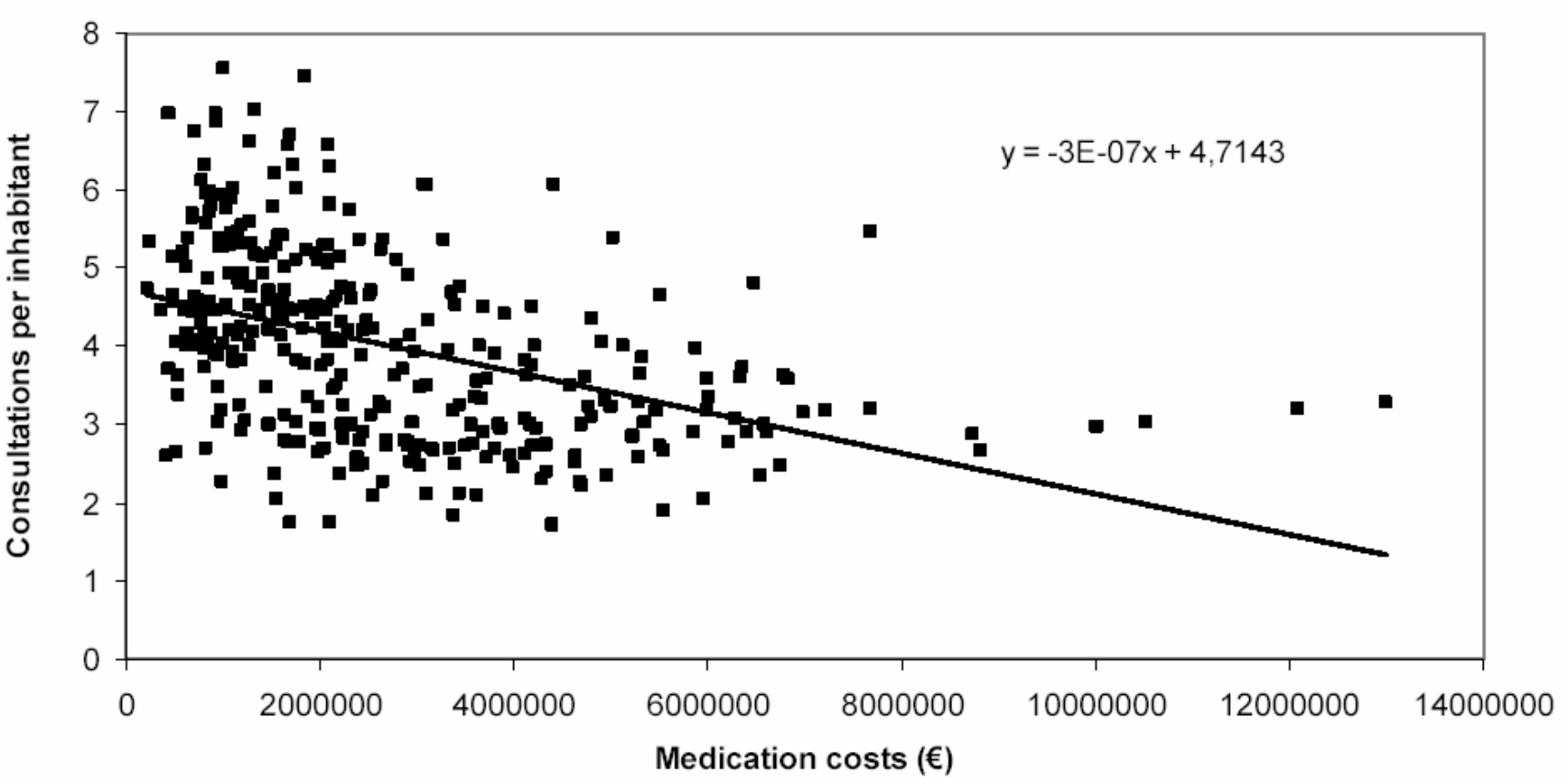
- Expenditure on drugs, supplies and equipment as inputs in the production function of workload weighted units (WWU) of several types of health care (medical, psychiatric, surgical, nursing home, intermediate care and outpatient visits) (Sexton et al., 1989).

- Pina and Torres (1992) considered that medication costs were an input either in the production of average number of consultations per inhabitant, the average number of consultations per professional per working day, or the percentage of programmed consultations.

Examples

- Draper (2000) used inpatient expenses as an input for physician ambulatory encounters, non-physician ambulatory encounters, and the inverse of hospital inpatient days. Inpatient expenses may have an inverse correlation with the inverse of hospital inpatient days.

Evidence from Portuguese data



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